MATHEMATICAL MODELLING OF THE FORMANT STRUCTURES OF VOCALIC SOUNDTYPE SYSTEMS

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ABSTRACT

Some problems of the formant structure modelling of vocalic soundtype systems (VSTS) by methods of multidimensional analytic and descriptive geometries as well as theories of convexes and inequalities are treated.

GENERAL CONCEPT

A concept of the acoustic structure modelling of VSTS was presented in a most general form formerly. By modelling we understand in this case a multistage process involving diverse aspects of the formant structure transfer of VSTS through mathematical structures and their graphic representation. Philosophy of modelling has exhaustively been considered elsewhere. Here, some basic problems, pertaining to principal modelling stages, are examined.

MODELLING AS A PROCESS

Sampling

The most effective acquisition of formant frequencies is to accomplish in a computer's memory coupled with an automatized formant frequency extraction yielding high precision readings. This stage deals also with statistical estimates of the formant data derived and with an evaluation of representing centroids (centers of gravity) of soundtypes as well. Measuring formant frequencies in spectrograms causes errors, is quite laborious and should preferably be avoided. However, problems of linguistic selection and phonetic realization of samples have undoubtedly to prevail at this stage.

Option and Construction of Models

There are three special kinds of modelling the F-structures of VSTS, producing correspondingly three types of models. Option of a particular model type depends on its purpose. Thus, the typology of the models in question covers the following types: (I) models of single soundtypes and of their systems through single models formed with approximating polinomials of 1st and 2nd degree; (II) models of VSTS formed with vector-to-point soundtype representation; (III) models of VSTS formed through axonometric constructions.

The modelling consists in formation of closed convex images in a multidimensional modelling space under employment of geometrical methods. In principle, a topological approach is also possible. However, geometrical constructions are important means of activating and stimulating the intuitive euristic image-bearing thinking.

Let us introduce a formant space of n dimensions with the Euclidean metrics therein. Then, the distance between two soundtypes X and Y with the formant frequency values X('F1, 'F2, ...'Fn) and Y ("F1, "F2, ..."Fn) is expressed as

$$L=(('F1-"F1)^2+('F2-"F2)^2+ \dots + ('Fn-"Fn)^2)^{1/2} \qquad (1)$$

The modelling F-space is necessarily isometrical if the coordinate axes therein are linearly scaled. With this goal in mind, both the natural frequency values as well as their logarithms linearly scaled are applicable. Since the image clarity and the complicacy of models are conflicting claims, subspaces of less than n dimensions are to be introduced. Thus, introducing, for instance, subspaces of 2 dimensions in the F-space of n dimensions, we have P subspaces which are actually modelling F-hyperplanes:

$$P= \sum_{q=1}^{q=n-1} (n - q). \qquad (2)$$

Single Soundtype Models

These models as well as such of VSTS through single models reflect the distribution of formant frequencies in the modelling F-space under condition of plural realization of soundtypes. Construction of models consists in an adequate linear or/and unlinear approximation of soundtypes in the F-space with closed convexes and in working out equivalent sets of linear or/ /and unlinear inequalities.

Linear Approximation. The problem of linear approximation of soundtypes in the modelling F-space consists in forming convex areas by means of hyperplanes of less dimensions than the F-subspaces are in compliance with the formant data of soundtypes. Hence, e.g. a 3-D F-space contains, in accordance with (2), three 2-D F-subspaces and 0-D, 1-D hyperplanes as simplexes embodied in points and vectors, s. Table 1. Apparently, simplexes may be formed with hyperplanes of no more than n-1 dimensions, cutting out the simplexes in the F-space. Any hyperplane can be described then as

$$a_1F_1+a_2F_2+\dots$$
$$+\dots a_{n-1}F_{n-1}+a_nF_n+a_{n+1}\geqslant 0 \quad (3)$$

When reffering to modelling 2-D F-subspaces and 1-D hyperplanes, a model will be formed with 1-D simplexes given through algebraic sets comprising inequalities of the following form:

$$a_{11}F_k+a_{12}F_{k+1}+a_{13}\geqslant 0. \quad (4)$$

Forming a v-dimensional simplex of (v-1)-dim. hyperplanes, a minimal number of hyperplanes, forming the simplex, can be written in the form

$$h_{min}\geqslant v+1 \quad (5)$$

The coresponding number of inequalities, describing this soundtype, amounts then to:

$$H_{min}\geqslant P(v+1) \quad (6)$$

The sign of inequality in (3) and (4) indicates the sharing of the F-space into two F-subspaces: the polynomial has negative values in one F-subspace and positive ones or zero in the other. A set of inequalities, in this way, is capable of describing a complex of limited convex F-subspaces, forming a polyhedral (if v=2, then it is a polygonal domain) closed convex area of solutions of the set modelling a soundtype given. Obviously, a more extended set of modelling inequalities contributes to a better approximation of soundtypes.

Unlinear Approximation. This type of approximation is based on the formation of n-1 dimensional closed convex (hyper)surfaces of the second degree, or quadrics, to be described through quadratic inequalities and enclosing the point sets, corresponding to soundtypes given, in the F-space of n dimensions. It includes several conoidal types: hyperboloidal, paraboloidal, and ellipsoidal approximations. Preferably, an ellipsoidal approximation should be used as yielding closed convex hypersurfaces. However, this procedure is rather laborious, so that a meaningful use of 2-D subspaces, consistently with (2), instead of the n-dim. modelling F-space has to be preffered. The quadrics used will have then the index 1-D, and the approximation of soundtypes will make use of second degree curves: 1-D ellipsoids (ellipses) or even 1-D spheres (circles). Pascale and Brianchon's theorems are helpful when constructing approximating ellipses. Parabolic or hyperbolic approximations are also applicable.

A second degree inequality in a most general form is as follows:

$$\sum_{i,k=1}^{n} a_{ik}F_iF_k + 2\sum_{i=1}^{n} b_iF_i + C\geqslant 0 \quad (7)$$

First, it can be reduced to the form

$$w_1{'F_1^2}+w_2{'F_2^2}+w_3{'F_3^2}+\dots$$
$$\dots+w_n{'F_n^2}+w_{n+1}\geqslant 0 \quad (8)$$

Table 1

ELEMENTARY LINEAR MODELLING CONSTRUCTS (SIMPLEXES)

Number of soundtypes in simplex	Simplex	Dimension of simplex	Formal characteristic parameters
1	Point	0	-
2	Vector	1	Vector length, distance between vector's ends
3	Triangle	2	Side length, area, angles, gravity center position
4	Tetrahedron	3	Edge length, side area, volume, gravity center position

Table 2

UNLINEAR CLOSED CONVEX IMAGES APPROXIMATING VOCALIC SOUNDTYPES

Modelling geometrical image	Dimension of modelling F-(sub)space	Approximating inequality in general form	Formal characteristic parameters
Ellipse	2	$a_{11}F_k^2+2a_{12}F_kF_{k+1}+a_{22}F_{k+1}^2+$ $+2a_{13}F_k+2a_{23}F_{k+1}+a_{33}\leqslant 0$	Half-axis length, contraction factor, area, gravity center position
Ellipsoid	3	$a_{11}F_1^2+a_{22}F_2^2+a_{33}F_3^2+2a_{12}F_1F_2+$ $+2a_{23}F_2F_3+2a_{13}F_1F_3+2a_{14}F_1+$ $+2a_{24}F_2+2a_{34}F_3+a_{44}\leqslant 0$	The same; volume

When reffering to 2-D and 3-D modelling (sub)spaces, (8) is reduceable to the forms indicated in Table 2. In the case of an n-dimensional ellipsoidal approximation, we have:

$$\frac{'F_1^2}{t_1^2}+\frac{'F_2^2}{t_2^2}+\frac{'F_3^2}{t_2^2}\dots+\frac{'F_n^2}{t_n^2}-1\leqslant 0 \quad (9)$$
$$t_n^2=+(w_{n+1}/w_n)$$

Plane VSTS Models

Structural models of VSTS are constructed by means of elementary linear constructs (s. Table 1) when higher forms of abstraction are substituted for lower ones. It is natural to construct a structural model through a transformation of single soundtype models and by reducing them to 0-D simplexes in the n-dim. modelling space. This implies that higher dimension simplexes of single soundtype models are substituted by lower dimension simplexes of structural models. These latter incorporate into sets of higher dimension simplexes, thus forming spatial models of the F-structures of VSTS. It is obvious that in 2-D (sub)spaces the use of modelling simplexes of 0 to 2 dim. and in 3-D (sub)-spaces that of simplexes of 0 to 3 dim. is possible. In any case, a structural model is cut out through hyperplanes as a multidimensional polyhedron with every vertex representing a single soundtype. A geometrical interpretation of the F-structures of VSTS and formal characteristic parameters as well allow some acoustic problems in the n-dim. F-space to be solved by means of numerical and graphometrical methods.

Axonometric VSTS Models

Axonometric models are essential for a spatially condensed picturelike portraying of the F-structures of VSTS in the n-dim. F-space (n 3). Though the problem of visual support through graphic aids in science is rather vague, it is not reasonable to underestimate a contribution of image-bearing thinking and euristic factors in research work and education. However, the solution of metric problems with axonometric models becomes too complicated so that their use is limited by illustrative and demonstrative goals. Also, there are some specific problems in application of multidimensional descriptive geometry methods to the modelling of the F-structures of VSTS. Rather promising in this respect seems to be the construction of axonometric models on the basis of a pair of usually available, reciprocally orthogonal, plane modelling images.

VERIFICATION AND IDENTIFICATION OF MODELS

These modelling stages are required to prove the agreement between the soundtypes to be modelled and their models. The modelling process is considered as being completely finished if only the following chain has been preserved: (a) soundtype X, (b) model A, B..., (c) soundtype Y... The relationships between the links of the chain are of fundamental importance and manifest themselves in the course of the verification of the dyad (a) and (b) or/ and the identification of the dyad (b) and (c). As a result, an agreement or a disagreement between the soundtypes X and Y can be stated, the relationships between the soundtypes and the models being supposed those of a structural analogy.

Principally, the following relationships between soundtypes and their models are to be expected: (I) reflexivity, (II) transitivity, (III) antisimmetry. These qualities of a model are usually combined with each other, but if being absent, they imply the presence of a converse quality as it will be clearly shown below. The model qualities mentioned above signify the following: (I) a soundtype is a model of its own; (II) a model's model is a model of the prototype; (III) a model in general is a homomorphic image of a soundtype given and can be substituted for the latter within the limits of its characteristics of significance which permit to state a structural analogy. Thus, verification and identification both are counterpart processesand qualify the relationships between soundtypes and their models as follows: (i) b:a=a:b, b:a≠a:b (verification, antisimmetry/simmetry); (ii) b:c=c:b, b:c≠c:b (identification, antisimmetry/simmetry; (iii) if b:a=a:b and b:c=c:b, then a=c (transitivity/antisimmetry)

In this way, 1) if the model A is separately adequate to soundtypes X and Y, then X=Y (transitivity, antisimmetry); 2) if the model A is not adequate to at least one of soundtypes X and Y, then X≠Y (absence of transitivity, simmetry); 3) if the model A is adequate to a soundtype X while the model B is adequate to a soundtype Y, and A=B, then X=Y (reflexivity, transitivity, antisimmetry); 4) if in the preceding item A≠B, then X≠Y (absence of transitivity, simmetry).

Summing up, we may state that the above-mentioned relationships as well as the deformations of models are subject to investigation by means of:A) characteristic parameters (s. Tables 1 and 2); B)geometrical affine transformations of the models /including 1) parallel tarnsfer of the F-structure in the F-space, 2) rotation of the F-structure in the F-space; 3)contraction or expansion of the structure along the coordinate axes in the F-space/.

SUMMARY

A brief account of means and ways of the mathematical modelling of the acoustic structures of vocalic soundtype systems by methods of multidimensional analitical and descriptive geometries, theories of convexes and inequalities has been presented. The modelling stages may well involve the use of computers and graph plotting devices as working tools. In general, the geometrical approach traced proves to be an effective means of the mathematical modelling of vocalic soundtype systems in research work, demonstration and illusration processes.

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